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Research Statement

I am very interested in mathematical physics - or physical mathematics, because of the rich structures that often appear as the perspectives of pure mathematics interlace with those of theoretical physics. Physical intuition is often a beacon for mathematics, and reciprocally, some fields in pure mathematics can guide physics. In the following I present my contributions to some research topics on which I have been working or/and am still working on: the monstrous moonshine correspondence, brane tilings and dynamical supersymmetry breaking in string theory, and (cluster) higher-Teichmüller theory. All these research topics lie at an interface between theoretical high-energy physics and mathematics. A deep desire of mine as a middle-term goal is to manage to make good use of the different perspectives from which they can be approached.

Past research

Hecke modular groups and bipartite graphs I have been working with Prof. Yang-Hui He, at City, University of London, on some intriguing coincidences between the number of cusps of the modular curves appearing in the moonshine correspondences for the three biggest sporadic groups, and exceptional Lie algebras. The 15 genus zero quotients of the upper-half plane by Hecke congruence subgroups of the modular group $\mathrm{PSL}_2(\mathbb{Z})$ provide a nice toy example of this: these complex curves have 56 cusps in total, and the sum of the squared number of cusps over all 15 curves is $266 = 2 \times 133$, which strongly suggests some link with the exceptional algebra E_7 (since its fundamental representation is of dimension 56, while its adjoint is 133-dimensional).

Even if we have not been able to explain this connection yet, together with Y.-H. He and J. McKay we have developed a new approach to Hecke congruence groups based on an intuitive definition of arithmetic groups proposed by Conway in [Con96] and on the algebraic bipartite maps of Grothendieck's dessins d'enfants [Gro13]. It provides new elementary ways to compute of quantities related to the Hecke congruence groups, such as the genus, the number of cusps and torsion points, together with a fine understanding of the structure of the quotient modular curves. Our results, together with a tabulation of the 15 dessins d'enfants corresponding to the Hecke congruence groups of genus zero of our initial interest, are presented in [THM20]. I have also written a short and elementary introduction to the Monstrous Moonshine [Tat19] aiming to explain the ideas of the correspondence is a very pedagogical way.

Dynamical supersymmetry breaking and string theory It has appeared during the 20th century that quantum field theories (QFTs) and in particular gauge theories form the right framework to describe elementary particles and their interactions. Decades of intense model building motivated by numerous experimental discoveries culminated with the construction of the Standard Model of particle physics in the 70's. The rate of experimental breakthroughs decreased shortly after that and new types of questions arose in theoretical high-energy physics, of a more conceptual nature. Some of these questions were related to special features and the consistency of the Standard Model, others to the general behaviour of QFTs at strong coupling.

A property of QFTs known as supersymmetry (SUSY) proved very fruitful regarding these matters: supersymmetric extensions of the Standard Model often provide an explanation for the hierarchy problem, improve the convergence of the gauge coupling constants at the Grand Unification Scale, and contains natural candidates for particles constituting dark matter. Moreover, among all QFTs the supersymmetric ones are much more tractable, since SUSY implies numerous non renormalisation theorems and techniques which allow to peep at strong coupling dynamics of the theory and sometimes even understand it thoroughly. SUSY is intrinsically related to deep mathematical structures such as Kähler, hyperkähler and algebraic geometry.

Despite all the nice features of SUSY, our low-energy universe is manifestly not supersymmetric, and asking for such a symmetry at high energies requires an explanation of how it breaks spontaneously at low energies. Implementing the spontaneous breaking of SUSY in an interesting way is notoriously difficult, the simplest models demanding an auxiliary QFT dubbed hidden sector. In order to explain an exponential hierarchy between the Planck scale and the SUSY breaking scale, SUSY must be broken by dynamical effects in the hidden sector [Wit81]. When this is the case is a SUSY QFT one speaks of Dynamically Supersymmetry Breaking (DSB) models.

A fistful of DSB models are known since the 80's [ADS85] – in what follows the theories known as uncalculable SU(5) model and $3-2$ model (together with their generalizations) play a special role.

Type II superstring theories contain particularly interesting solitonic objects known as D-branes and NS5-branes. D-branes host gauge fields and can be arranged in setups from which one engineers and studies various aspects of supersymmetric QFTs. Moreover one can also introduce orientifold planes in these setups, and that enlarge even more the set of QFTs one can obtain in brane configurations. D-branes are also at the heart of holographic dualities between supergravity theories and QFTs, the first explicit instance of such a duality being Maldacena's celebrated AdS/CFT correspondence [Mal99]. Putting a large number of coincident D3 branes at the tip of an affine toric Calabi-Yau threefold (CY3) singularity in type IIB string theory, one obtains generalizations of the original AdS/CFT correspondence with a smaller number of supersymmetries or without conformal invariance if there are additional fractional branes at the singularity. The QFT on the worldvolume of the D-branes then exhibits a spectacular behaviour under the renormalisation flow known as a duality cascade, the supergravity dual of which has also been described as warped throats [KS00].

Brane tilings – introduced in [HK05, FHV$^+$06, FHM$^+$06] – describe configurations of D5 and NS5 branes in type IIB theory which are T-dual to a stack of D3 branes at a CY3 singularity - possibly with fractional branes. They are very efficient combinatorial tools to derive the D3 brane worldvolume QFT. A dictionary between the geometry of the CY3 and the corresponding brane tilings has been established and enriched over the years. Prosaically, brane tilings are bipartite graphs embedded in a torus T^2 and satisfying consistency conditions such that they admit (possibly degenerate) isoradial embeddings [IU10].

Orientifolds can also be incorporated in brane tilings [FHK$^+$07] and enlarge the set of interesting QFTs they can engineer. In particular the DSB SU(5) and $3-2$ models can be embedded in brane tilings with orientifolds, letting one hope for a possible construction of non-supersymmetric vacua in a local model which would naturally provide a UV completion of these theories. However, it was shown more recently in a specific example [BGVU19] that in this case the UV completion spoils the non-supersymmetric vacuum because the singularity under consideration is not-isolated, which results in a runaway direction in the moduli space of the UV-completed theory. A general no-go theorem was proved in [ABMP19], asserting that UV completions by brane tilings indeed always make the non-supersymmetric vacua unstable as soon as the singularity under consideration is not isolated – equivalently, when there exist special fractional branes known as $\mathcal{N}=2$ fractional branes at the singularity.

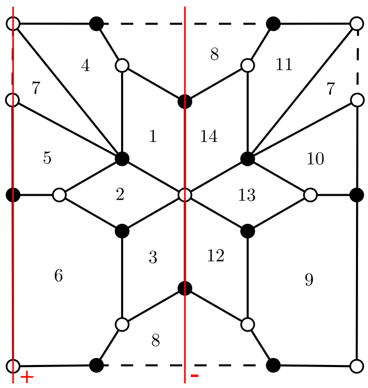

Figure 1: The octagon.

An extensive search for the SU(5) and $3-2$ models in "small" isolated affine toric CY3 singularities and finding no instance at all from this search [ABMP19] led to the conjecture that it is impossible to embed such theories in a brane tiling without $\mathcal{N}=2$ fractional branes - and to a swampland-like conjecture about the infeasibility of engineering DSB models in string theory. Together with Riccardo Argurio, Matteo Bertolini, Sebastián Franco, Eduardo García-Valdecasas, Shani Meynet and Antoine Pasternak we showed that all the structures possibly leading to the SU(5) or $3-2$ models in brane tilings indeed need $\mathcal{N}=2$ fractional branes, but one. We also managed to construct a brane tiling we dubbed the octagon, which describes a $d=4$, $\mathcal{N}=1$ QFT with 14 gauge groups containing a twin version of the SU(5) model on a deformation brane – and without $\mathcal{N}=2$ fractional brane [ABF$^+$21c, ABF$^+$21b]. As complicated as the octagon brane tiling may be, it is likely to be the smallest one that meets our requirements, for it it is the case the toric diagram corresponding to the singularity must have at least 8 sides (and this is exactly the case for the octagon).

Along the way we developed tools and criteria to understand under which conditions orientifolds of toric CY3 can correspond to non-anomalous brane tilings without flavor branes [ABF$^+$21a] – this is a new entry in the geometry/brane tiling dictionary.

Brane tilings form a rich laboratory to study aspects of brane setups in type II superstring theories, the gauge/gravity correspondence, $d=4$ $\mathcal{N}=1$ supersymmetric quantum field theories and phenomenology. Moreover through the use of these objects I have a developed a good understanding and technical knowledge of affine toric CY3; for instance I am familiar with the program Macaulay2 with which one can compute a Hilbert basis of the dual cone corresponding to a toric diagram and a minimal set of generators defining the affine variety. In order to construct the octagon presented in Figure 1 we have introduced the use of a combinatorial tool known as triple-crossing diagrams, which systematizes the inverse algorithm and allows to construct dimer models satisfying symmetry constrains or containing particular substructures. I have succinctly reviewed these techniques and some applications in [Tat21].

With Eduardo García-Valdecasas, Shani Meynet and Antoine Pasternak we have studied general aspects of the implementation of orientifolds in brane tilings by studying all five possible smooth involutions of the latter. We have shown that on top of the three types of orientifolds in brane tilings presented in [FHK$^+$07], glide reflections could also encode SUSY preserving orientifolds. This provides a new set of $d=4$ $\mathcal{N}=1$ conformal field theories, gives a physical meaning to dimer models on Klein bottles, and completes the classification of orientifolds of brane tilings [GVMPT21].

Higher Teichmüller theory and cluster varieties Another facet of the reasearch I have been doing up to today is about cluster algebras and varieties, and higher Teichmüller theory. These topics are traditionally pure mathematics topics however they lie at a very porous interface between mathematics and theoretical physics, as we shall see.

If S is a hyperbolic topological surface, the Teichmüller space $\mathcal{T}(S)$ of S is the set of hyperbolic metrics (equivalently, complex structures) S can be endowed with, up to the diffeomorphisms continuously connected to the identity. It is more than just a set, and enjoys various interesting features [Pap16]. Since the automorphism group of the hyperbolic disk is $\mathrm{PSL}_2(\mathbb{R})$, choosing a hyperbolic structure on S is equivalent to the choice of a discrete and faithful group morphism

$$\pi_1(S) \to \mathrm{PSL}_2(\mathbb{R}) \ , \tag{1}$$

modulo overall conjugation by $\mathrm{PSL}_2(\mathbb{R})$.

The theory has different guises depending on if S is closed or not. In the case where it is not, Penner [Pen87] and Thurston [Thu86] have introduced remarkable sets of coordinates on two versions of the Teichmüller space, respectively denoted $\mathcal{T}^a(S)$ and $\mathcal{T}^x(S)$. Both correspond to a triangulation of S. As the latter changes, coordinates undergo the mutation formulas for cluster variables and Y-patterns, respectively [FG07]. Cluster algebras have been defined by Fomin and Zelevinski [FZ02] as an abstraction of the exchange relations appearing in the study of double Bruhat cells in reductive algebraic groups [FZ99] motivated by Lusztig's theories of total positivity and dual canonical bases. A cluster algebra of rank n is a commutative algebra generated by a set of Laurent polynomials in n variables constructed from the mutation class of a quiver. Cluster varieties are the algebraic varieties corresponding to cluster algebras [FG09]. To each mutation class of a quiver one can associate two kinds of cluster varieties denoted $\mathcal{A}$ and $\mathcal{X}$. The Teichmüller spaces $\mathcal{T}^a(S)$ and $\mathcal{T}^x(S)$ with Penner and Thurston coordinates form an example of such a pair of cluster varieties.

In general $\mathcal{X}$ and $\mathcal{A}$ cluster varieties are respectively endowed with a Poisson structure (which admits a canonical deformation quantization) and with a (degenerate) closed 2-form (which admits a motivic avatar). Cluster varieties are positive varieties and hence one can study their points over arbitrary semi-fields such as $\mathbb{R}_{>0}$ or the tropical semi-fields $\mathbb{Z}^t, \mathbb{Q}^t, \mathbb{R}^t$. A bunch of dualities proved at least partially in [GHKK18] relates the tropical cluster variety of one type to the algebra of (universally Laurent) functions on the other. In the context of Teichmüller spaces, the $\mathbb{Z}^t$-points of the cluster varieties $\mathcal{T}^a(S)$ and $\mathcal{T}^x(S)$ can be interpreted as the spaces of rational laminations of two types on S.

It has proved interesting to generalize the Teichmüller spaces defined above in the following way: given a split real group $G(\mathbb{R})$, a $G(\mathbb{R})$-higher Teichmüller space of S is a connected component of the quotient of discrete and faithful group morphisms

$$\pi_1(S) \to G(\mathbb{R}) \ , \tag{2}$$

under conjugation by $G(\mathbb{R})$. Such spaces can be described in terms of Higgs bundles [Hit92]. Fock and Goncharov constructed higher Teichmüller spaces for $G(\mathbb{R})$ simply-connected or with trivial center, as the set of $\mathbb{R}_{>0}$-points of the corresponding framed $G(\mathbb{C})$-character varieties (which are cluster varieties) [FG06].

Some structures naturally related to classical Teichmüller spaces such as laminations do not generalize obviously in the higher setting. According to the cluster duality philosophy one can formally define integral $\mathcal{A}$ (resp. $\mathcal{X}$) higher laminations as the set of $\mathbb{Z}^t$-points of the corresponding $\mathcal{X}$ (resp. $\mathcal{A}$) cluster variety, however a truly combinatorial description of higher laminations in the spirit of classical laminations is still lacking. The spectral networks of Gaiotto, Moore and Neitzke [GMN13] introduced as a tool to study the BPS spectrum of $4d\ \mathcal{N}=2$ theories of class $\mathcal{S}$ seem to go in the right direction since a spectral network of type A_k on S provides a function on the space of flat $\mathrm{SL}(k,\mathbb{C})$-connections on S through abelianization. Moreover the connection between Hitchin integrable systems and the work of Seiberg and Witten on Coulomb branches of $4d\ \mathcal{N}=2$ theories is known since long [DW96]. However, there are too few functions coming from spectral networks with respect to what one expects. Instead of considering $(k+1)$-fold holomorphic covers of S as one does in M-theory to describe the Coulomb branch of class $\mathcal{S}$ theories of type A_k, one should rather consider k-fold Lagrangian covers of S inside T^*S. Such covers are in turn related to the Hecke algebra of the affine Weyl group $\widehat{W}_{A_k}$.

Given an ideal triangulation of S, one is led to assign basis elements of the Hecke algebra $\mathcal{H}(\widehat{W}_{A_k})$ to its edges in order to define higher laminations. With Vladimir Fock and Alexander Thomas we have noticed that if one replaces $\mathcal{H}(\widehat{W}_{A_k})$ with a Hecke algebra $\mathcal{H}(W)$ corresponding to a finite Coxeter system W one can associate a Laurent polynomial to the pair (S, W) in a natural way. The latter does not depend on the choice of triangulation and hence the construction is a formal topological quantum field theory. It turns out it is closely related to the representation theory of $\mathcal{H}$ [FTT21].

Cluster varieties do not only appear in higher Teichmüller theory: Goncharov and Kenyon have defined a class of integrable systems whose phase spaces are cluster $\mathcal{X}$ varieties [GK13]. These integrable systems are constructed from bipartite graphs on T^2 satisfying exactly the same consistency conditions as the brane tilings. Moreover, cluster mutations in the context of these integrable systems corresponds to "spider moves" of the bipartite graph, which are exactly the combinatorial transformations of brane tilings describing Seiberg dualities of the corresponding QFTs.

The $\mathcal{X}$-coordinates on the phase spaces of the integrable systems appear as coordinates on the master space of the corresponding QFT [AFM12].

Fock and Marshakov have constructed cluster integrable systems on affine Poisson-Lie groups, of which the dimer integrable systems of Goncharov and Kenyon are the special cases corresponding to the groups $\widehat{A}_n$ [FM16].

Future directions

Supergravity dual and compactification of the octagon dimer model The SU(5) DSB model admits a generalization as a DSB model with $\mathrm{SU}(M+4) \times \mathrm{SU}(M)$ gauge group for any odd M and prescribed matter fields [ADS85]. It seems that the twin version of this generalized model – which appears as we populate the octagon with more and more deformation fractional branes of the good kind – is still DSB. This let one hope for a supergravity dual description of dynamical supersymmetry breaking, with possibly many thrilling applications.

The singularity hosting the octagon is non-compact. While that makes the gauge/gravity duality cleaner, it is less phenomenological than a true compactification of string theory. I would like to study flux compactifications on a Calabi-Yau threefold containing the octagon singularity at some corner of the moduli space. This construction could have interesting phenomenological and cosmological implications as in [CGEQ+21].

Higher laminations and spectral networks I would like to continue investigating to find a combinatorial definition of higher laminations in the spirit of spectral networks. After this is done I would like to establish a dictionary between higher laminations and spectral networks, and apply the results, techniques and intuition from one field, to the other.

Dimer models and cluster integrable systems Another undoubtedly very interesting dictionary to establish is the one relating dimer models (or related objects, such a $5d$ BPS quivers) to cluster integrable systems. One can hope for a physical implementation of cluster dualities, the use of orientifolds in cluster integrable systems as a bridge between the construction of Goncharov of Kenyon and the one of Fock and Marshakov, and the study of the relationship between duality cascades and discrete integrable systems.

Miscellaneous I have studied (homological) mirror symmetry, topological string theories and the connections with mathematics they provide: for instance knot theory, and the mathematically rigorous definitions of branes in the A and B models through triangulated Fukaya categories and derived categories of coherent sheaves. There is also a cloud of notions related from a distance or close up to the topics I have been working on: Borel summation and resurgence which connects to cluster varieties through WKB approximation in the work of Gaiotto, Moore and Neitzke, brane webs, 5d gauge theories together with the recent developments on their Higgs branches with the appearance of affine Grassmanians, and higher-form symmetries.

I would enjoy a lot working more deeply on all these topics, though not necessarily less than discovering new fields in mathematical physics that I am not aware of, yet.

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